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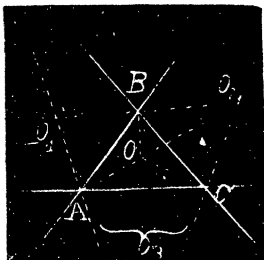
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II. Solution by WALTER H. DRANE, Graduate Student at Harvard University, and J. SCHEFFER, A. M., Hagerstown, Md.

Let ABC be the given triangle, O, O_1, O_2, O_3 , the centers of the inscribed and the three escribed circles. Then $O_1BO_2, O_1AO_3, O_2CO_3$, and AOO_2 are straight lines; also OB, OA, OC are each perpendicular to O_1BO_2, O_1AO_3 , and O_2CO_3 , respectively. In triangles AOC and BAO_2 , $\angle OAC = \angle BAO_2$ and $\angle OCA = \angle BO_2A$ since we have $\angle OCA = 90^\circ - \angle ACO_3 = 90^\circ - \frac{1}{2}(\angle CAB + \angle CBA) = 90^\circ - (\angle OAB + \angle OBA) = 90^\circ - [180^\circ - (\angle O_1AB + \angle O_1BA)] = 90^\circ - \angle O = \angle O_1O_2A$.



\therefore triangles AOC and ABO_2 are similar, and we have

$$AO : AB :: AC : AO_2 \dots \dots \dots (1).$$

Again in triangles O_1BA and AO_3C , $\angle O = \angle ACO_3$ and $\angle O_3AC = \angle O_1AB$. Hence triangles O_1BA and AO_3C are similar, and we have,

$$AO_1 : AC :: AB : AO_3 \dots \dots \dots (2).$$

Multiplying (1) by (2) $AO \cdot AO_1 : AB \cdot BC : AB \cdot AC : AO_2 \cdot AO_3$.

$$\therefore AO \cdot AO_1 \cdot AO_2 \cdot AO_3 = AB^2 \cdot AC^2.$$

Q. E. D.

III. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

In the figure of the last solution, draw the lines OD and O_1D_1 perpendicular to AC . Then $AO = \sqrt{(OD^2 + AD^2)} = \sqrt{(r^2 + r^2 \cot^2 \frac{1}{2}A)} = r \operatorname{cosec} \frac{1}{2}A$.

Similarly, $AO_1 = r_1 \operatorname{cosec} \frac{1}{2}A$.

$$AO_2 = \sqrt{(OO_2^2 - AO^2)} = AO \sqrt{[(OO_2^2/AO^2) - 1]} = AO \cot \frac{1}{2}C.$$

Similarly, $AO_3 = AO \cot \frac{1}{2}B$.

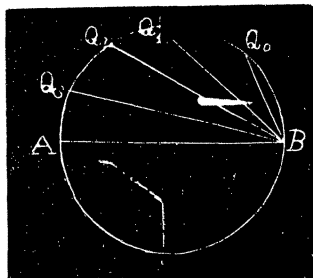
$$\begin{aligned} \therefore AO \cdot AO_1 \cdot AO_2 \cdot AO_3 &= r^3 r_1 \operatorname{cosec}^4 \frac{1}{2}A \cot \frac{1}{2}B \cot \frac{1}{2}C \\ &= (s-a)(s-b)(s-c) \operatorname{cosec}^4 \frac{1}{2}A \tan^2 \frac{1}{2}A \\ &= s(s-a)(s-b)(s-c) / \sin^2 \frac{1}{2}A \cos^2 \frac{1}{2}A \\ &= 4s(s-a)(s-b)(s-c) / \sin^2 A = 4S^2 / \sin^2 A. \end{aligned}$$

$$\text{But } \sin A = 2S/bc = 2S/AB \cdot AC. \therefore AO \cdot AO_1 \cdot AO_2 \cdot AO_3 = AB^2 \cdot AC^2.$$

Also solved by ELMER SCHUYLER.

101. Proposed by E. W. MORRELL, A. M., Late Professor of Mathematics, Montpelier Seminary, Montpelier, Vt.

AB is the diameter of a circle and Q_0 any point on the circumference; $Q_1, Q_2, Q_3 \dots$ are the points of bisection of the arcs $AQ_0, AQ_1, AQ_2 \dots$. Prove that $BQ_1, BQ_2, BQ_3 \dots BQ_n = OA^n \cdot (AQ_0/AQ_n)$.



Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; J. SCHEFFER, A. M., Hagerstown, Md., and ELMER SCHUYLER, High Bridge, N. J.

Let O be the center of the circle.

$$\angle ABQ_0 = \theta.$$

$$\therefore BQ_1 = AB \cos \frac{1}{2}\theta, BQ_2 = AB \cos(\theta/2^2).$$

$$BQ_3 = AB \cos(\theta/2^3), BQ_n = AB \cos(\theta/2^n).$$

$$AQ_0 = AB \sin \theta = 2AB \sin \frac{1}{2} \theta \cos \frac{1}{2} \theta = 2^2 AB \sin(\theta/2^2) \cos(\theta/2^2) \cos \frac{1}{2} \theta \\ = 2^n AB \sin(\theta/2^n) \cos \frac{1}{2} \theta \cos(\theta/2^2) \dots \cos(\theta/2^{n-1}).$$

$$AQ_n = AB \sin(\theta/2^n).$$

$$\therefore (AQ_0/AQ_n) = 2^n \cos \frac{1}{2} \theta \cos(\theta/2^2) \dots \cos(\theta/2^{n-1}).$$

$$BQ_1 \cdot BQ_2 \cdot BQ_3 \dots BQ_n = (AB^n) \cos \frac{1}{2} \theta \cos(\theta/2^2) \dots \cos(\theta/2^{n-1}) \\ = (\frac{1}{2} AB)^n (AQ_0/AQ_n) = (AO)^n (AQ_0/AQ_n).$$

CALCULUS.

78. Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

Investigate value of $\left(\frac{\tan x}{x}\right)^{1/x^n}$ where x is 0 and n has consecutive values 1, 2, 3, 4, Is there any law governing the different results? When $n=1$, result is 1; when $n=2$, result is $e^{\frac{1}{2}}$; $n=3$, gives ∞ , etc.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

$$\left(\frac{\tan x}{x}\right)^{1/x^n} = e^{(1/x^n) \log[(\tan x)/x]} = y.$$

$$\text{Limit of } \frac{\log \tan x - \log x}{x^n} = \text{limit of } \frac{\cot x \sec^2 x - (1/x)}{nx^{n-1}} = \text{limit of } \frac{2x - \sin 2x}{nx^n \sin 2x},$$

$$\text{but } \sin 2x = 2x - \frac{8x^3}{6} + \frac{32x^5}{120} - \frac{128x^7}{5040}, \text{ etc.}, = 2x - \frac{4x^3}{3} + \frac{4x^5}{15} - \frac{4x^7}{45} +, \text{ etc.}$$

$$\therefore \frac{2x - \sin 2x}{nx^n \sin 2x} = \frac{2x - 2x + \frac{4x^3}{3} - \frac{4x^5}{15} + \frac{4x^7}{45} -, \text{ etc.}}{nx^{n+1}(2 - \frac{4x^2}{3} + \frac{4x^4}{15} - \frac{4x^6}{45} +, \text{ etc.})} = \frac{30 - 6x^2 + 2x^4}{nx^{n-2}(45 - 30x^2 + 6x^4)}, \text{ ap-}$$

$$\text{proximately, } = \frac{2}{3nx^{n-2}} + \frac{14}{45nx^{n-4}} +, \text{ etc.}, = S.$$

When $n=1$, $S=0$ for $x=0$.

When $n=2$, $S=\frac{1}{2}$ for $x=0$.

When $n=3, 4, 5$, etc., $S=\infty$ for $x=0$.

\therefore When $n=1$, $y=e^0=1$.

When $n=2$, $y=e^{\frac{1}{2}}$.

When $n=3, 4, 5$, etc., $y=e^\infty=\infty$.

Also solved by ELMER SCHUYLER, whose solution has been accidentally misplaced, and hence does not appear in this issue.